

METHODOLOGY AND DATA COMPANION

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Scale Economies and the Cost of the Dual Curriculum

Methodology and data companion to the working paper

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A full white paper, with the complete introduction, related literature, data and design, results, discussion, limitations, conclusion, and references, is available on request.

© 2026 Heshbon and the author. This companion explains how the working paper was built and how to read its results. It can be shared and cited with attribution.

At a glance

Research question	Does running two instructional programs raise a school's per-pupil cost, and does enrollment growth lower Jewish day schools' per-pupil costs more than it lowers other schools'?
Data	National Association of Independent Schools benchmarking records (the DASL platform) for 1,163 PK-8 independent schools, with a principal cross-section of 497 schools reporting fiscal 2024 to 2025 operating expenses.
Unit of analysis	The individual school, and the school-year in the pooled and within-school designs, rather than a student or classroom.
Statistical method	A sector-interacted cost-curve regression (log expenditure per pupil on enrollment), assessed with a randomization (permutation) test.
Headline finding	At equal size and region, Jewish schools employ about 2.5 more full-time staff per 100 students and spend about 17 percent more per pupil; their costs appear to keep falling with size, but that slope difference does not clear rigorous inference.
What it shows	The level premium is real and robust; the slope advantage is a well-supported hypothesis that awaits more reporting schools.

The question behind the paper

A dual-curriculum school runs two academic programs, general studies and Judaic studies, in every grade it serves. That structure suggests two testable claims about cost. The first is a claim about levels, that such schools spend more per pupil at any size because parallel tracks require more staff. The second is a claim about slopes, that because the fixed cost of staffing a grade structure is roughly doubled, savings from growth should run deeper and last longer, so per-pupil costs keep falling past the point where single-curriculum schools flatten out.

The paper tests both and reaches a split verdict. The level claim is confirmed. The slope claim is supported by the point estimates in every design the author tried, yet it does not survive the

paper's chosen standard of statistical evidence, and that tension is itself part of the story.

Where the data come from

The benchmarking platform. All figures come from the National Association of Independent Schools' Data and Analysis for School Leadership platform, the sector's voluntary benchmarking survey. The peer group is 1,163 PK-8 schools, with each year's dollars paired to that year's enrollment. The principal cross-section is the 497 schools reporting fiscal 2024 to 2025 operating expenses with at least 20 students.

The analysis samples. A staffing sample of 471 schools adds full-time-equivalent counts, a within-school sample of 396 observes the same schools in two consecutive years, and a pooled

sample stacks both years into 976 school-year observations.

Identifying Jewish schools. Jewish day schools are flagged by a name-pattern screen (markers such as Hebrew, Torah, yeshiva, and Schechter) followed by manual review, giving 74 schools, of which 24 report fiscal 2024 to 2025 financials. Because the screen misses Jewish schools without such name markers, and any it misses stay in the comparison group, the measured contrasts are conservative. The classification is built so that a roster-based flag could later replace the name screen with no other change.

The statistical method, named and explained

The core tool is a cost curve, an ordinary least squares regression of log expenditure per pupil on enrollment (entered in a flexible quadratic form), interacted with an indicator for Jewish day

schools so the two groups can differ in both level and slope. Census-region fixed effects hold geography constant, and an accounting decomposition splits any gap into a staffing channel and a dollars-per-employee channel. A within-school first-difference design, which compares each school to itself a year earlier, removes fixed institutional differences and provides a second identification strategy. Because the treated group is small, 24 schools with a few at extreme enrollments, conventional robust standard errors can overstate significance in exactly this configuration, so the paper adopts a randomization (permutation) test as its standard of evidence: the Jewish label is reassigned at random to 24 schools 2,000 times, holding the regional distribution fixed, which builds the null distribution directly rather than from a formula. The divergence between the conventional and randomization results is reported openly and treated as a finding in its own right.

Key results from the paper

Table 1. Sample medians by sector, fiscal year 2024 to 2025 (staffing sample).

Sector	Schools	Median enrollment	FTEs per 100 students	Expense per pupil
Comparison	450	229	21.3	\$27,682
Jewish	21	311	23.8	\$32,497

Note. Each entry is the median school in its group. The Jewish schools employ about 2.5 more full-time-equivalent staff per 100 students and spend about 17 percent more per pupil, at ordinary class sizes and comparable faculty seniority.

Table 2. Elasticity of expenditure per pupil with respect to enrollment, by sector.

Enrollment	Comparison schools	Jewish schools	Interpretation at this size
100	-0.10	+0.10	comparison schools still gaining from growth
150	+0.01	0.00	both curves approximately flat
250	+0.14	-0.11	comparison rising, Jewish still declining
400	+0.27	-0.22	at 10% growth, comparison +2.7%, Jewish -2.2% per pupil
600	+0.38	-0.32	divergence widest, few schools this large

Note. Point estimates from the quadratic sector-interacted model, $N = 497$. Negative values mean growth continues to lower per-pupil cost.

Table 3. The joint sector-interaction test under four inference methods (p-values).

Design	Classical	Robust (HC1)	Robust (HC3)	Randomization
Expenditure curve, FY2024 to 2025 (24 Jewish)	0.20	0.002	0.032	0.23
Staffing curve, quadratic (21 Jewish)	0.55	0.14	0.28	0.53

Within-school slope (18 Jewish)	0.10	0.004	0.014	0.18
Pooled two-year curve (41 Jewish)	0.039	0.001	n/a	0.19

Note. Each entry is the probability of a sector difference at least as large as observed under the null of no difference. Randomization tests use 2,000 region-stratified label permutations. The pooled robust value uses a school-clustered covariance, and the pooled HC3 cell is not reported.

How to read the results

Table 1 carries the confirmed half of the paper. At the same size and in the same region, the typical Jewish school employs more people and spends more per pupil, and the class-size and seniority figures show the extra spending buys additional teachers rather than higher pay, which is the staffing signature of two parallel tracks.

Table 2 shows the slope pattern that motivates the paper. Comparison-school costs bottom out and turn upward as enrollment grows, the textbook U-shape, while the fitted Jewish curve keeps descending across the same range. Table 3 is where honesty about inference matters. Conventional robust standard errors reject identical curves decisively, yet the randomization test, which stays reliable when one group is small and holds influential schools, does not. Pooling two years moves the classical p-value from 0.039 toward significance while the randomization value stays at 0.19, so the evidence is drifting toward the hypothesis without arriving.

What the numbers do not claim

The paper is careful not to overstate the slope result. With only a few dozen Jewish schools reporting financial data, a pattern of this size arises by chance in roughly one random sample out of five, so the fair summary is that the hypothesis looks right and is not yet proven. Robustness checks make the fragility explicit: limiting the comparison to the 150 to 600 student range removes the curve-shape difference entirely, and dropping the three Jewish schools with the most extreme enrollments moves the conventional p-value from 0.002 to 0.10. The level premium, by contrast, does not depend on any single subgroup. Expenditure is also not cost at a fixed program, so the premium bundles the dual curriculum with anything else these schools choose to purchase, although the class-size and seniority results rule out the two most natural alternatives.

Reproducing the analysis

Every estimate derives from school-level extractions of the DASL Create Report exports for the All PK-8 peer group, with financial figures dated to the fiscal year they describe. Randomization tests use 2,000 region-stratified label permutations under a fixed random seed, and the pooled test reassigns labels at the school level so both of a school's years move together. The single clearest path to a firmer answer is broader financial reporting by Jewish day schools themselves, since only about one in three identified Jewish schools reports financials in a given year; on the patterns here, roughly forty more reporting schools would likely be decisive. Replication code and the analysis dataset are available from the author.